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# MODELLING OF STREAM FLOW AT SLAPTON WOOD USING TOPMODEL WITHIN AN UNCERTAINTY ESTIMATION FRAMEWORK

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## ABSTRACT

This paper presents the results of a streamflow modelling exercise for Slapton Wood catchment using TOPMODEL (Beven & Kirkby, 1979; Beven, *et al.*, 1995) within an uncertainty framework. For the period 1989 to 1991, flow at the main weir was modelled from rainfall measurements and evaporation estimates from the Institute of Hydrology's automatic weather station sited in the catchment. The hydrological record was split into four periods; a calibration period in late 1990 and three validation periods. The Generalised Likelihood Uncertainty Estimation (GLUE) technique of Beven & Binley (1992) was used to estimate uncertainty bounds on flow predictions for the three validation periods, conditioning the predictions on likelihoods derived from the calibration period. Observed flows in the validation periods are enclosed by the uncertainty bounds for most of the time. However, there was some evidence that flow prediction performance was hampered by the effects of structural changes to the model introduced in an attempt to model spatially distributed soil moisture.

## INTRODUCTION

The Slapton Wood catchment has been the subject of hydrological investigation since 1969, (e.g. Troake & Walling, 1973; Ratsey, 1975; Burt *et al.*, 1983; Trudgill, 1983; Burt & Arkell, 1987; Burt, 1987; Heathwaite *et al.*, 1990) and was intensively instrumented in 1989/90 by the Institute of Hydrology for a model validation exercise carried out by Newcastle University for Nirex UK Ltd. Because of the wealth of data this provided, together with the apparent suitability of the site for the application of TOPMODEL, the site was chosen for an investigation into the use of remote sensing and other internal state data in the calibration of the hydrological model TOPMODEL (Fisher, 1995). Work presented here concentrates on flow modelling within an uncertainty framework. The GLUE procedure of Beven & Binley (1992) was adopted whereby uncertainty in model parameterisation is represented as uncertainty bounds on flow predictions.

## MODEL DESCRIPTION

TOPMODEL has a simple but mathematically elegant conceptual basis which allows distributed predictions of hydrological processes whilst maintaining parametric and computational efficiency. The computational efficiency is achieved by employing a similarity approach, whereby spatially distributed phenomena are represented within the model as distribution (or probability density) functions. One of the most important such hydrologically relevant characteristics is the topographic situation of a point within the catchment. In TOPMODEL, this is quantified in terms of a topographic index, ln(a/tan) value (hereafter referred to as the ATB value), where a is the upslope contributing area per unit contour length and is the local slope angle (see Beven & Kirkby, 1979, Beven *et al.,* 1995, Quinn *et al.,* 1995). Within TOPMODEL the ATB value acts as an index of hydrological similarity; all points in the catchment with the same value of the index are predicted as responding in the same way. This greatly simplifies the calculations and reduces the computational cost while still allowing distributed predictions of soil moisture and water table levels to be made.

The topographic index for the Slapton wood catchment was derived automatically using a multiple flow direction algorithm program (Quinn *et al.,* 1991, 1995). The analysis utilised a digital elevation map (DEM) which was created by digitising contour information for the catchment from a 1:10560 scale topographic map and interpolating to a regular grid with a resolution of 10m. The spatial pattern of the index is shown in Fig. 1, superimposed on the topography for the Slapton catchment. High values of the index, indicating a tendency towards wetter conditions, are found in the valley bottom, in the three convergent (high *a* value) hollows, and in some places near to the divides as a result of the low hydraulic gradient as represented by the slope, tan. For use in TOPMODEL the distribution of the topographic index within the catchment is converted to a probability density function which is discretised into a number of ATB classes (see Fig. 2).

The model operates local root zone and unsaturated zone stores within each ATB increment, as shown in Fig. 3. The version of TOPMODEL used in this study is based on that formulated by Sivapalan *et al.* (1987) in which storage within the catchment is quantified in terms of depths to the water table. Assuming a constant transmissivity function within the catchment, local water table level for each ATB increment ($z_i$) is calculated from the catchment average water table depth ($\bar{z}$) according to the deviation of the local ATB value from the catchment average as follows:

$$z_i = \bar{z} - \frac{1}{f}\left[\lambda - \ln\frac{a}{\tan\beta}\right] \tag{1}$$

where $\lambda$ is the catchment average value of the $\ln(a/\tan\beta)$ index and f is the SZF parameter—the parameter of the assumed exponential decline of lateral transmissivity with depth into the soil profile and controls the form of the baseflow recession.

The local water table depth controls the amount of unsaturated zone storage, SUZ (see Fig. 3) for a given ATB increment and hence allows prediction of the spatial distribution of saturation in the catchment. Areas where the soil is predicted as being saturated may then generate saturation excess runoff. This will occur first in high ATB value classes which tend to be in the valley bottom and hillslope hollows. Additionally the model has been modified so that evapotranspiration can be supplemented from capillary rise dependent on the depth of the groundwater so that evapotranspiration and surface soil moisture are also modelled with a spatially distributed component. This was achieved by incorporating a layered root zone and limited recharge from groundwater to the root zone when the capillary fringe intersects the root zone (*i.e.* for shallow water table conditions). Fig. 2 shows schematically the way the ATB value varies with downslope position and how this results in a decrease in water table depth until surface saturation is reached, at which point saturation excess runoff is possible. By adopting the similarity approach model execution is limited to the number of classes within the

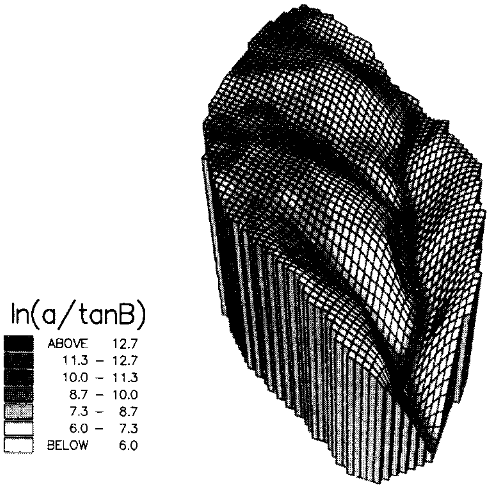

FIG. 1
ATB map for Slapton Wood Catchment

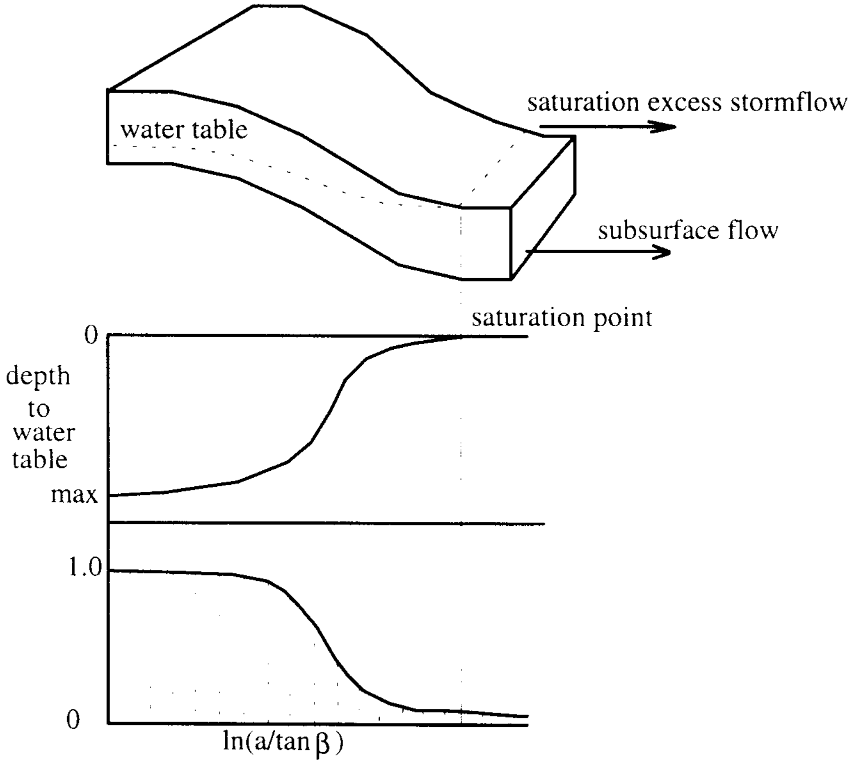

FIG. 2
Schematic of the TOPMODEL topographic index concepts showing the relationship between topographic index value on a slope, predicted water table depths, and partitioning of surface/subsurface flow generation.

distribution whilst making predictions for all cells within the catchment. Implicit in this approach is the assumption that all areas of equal topographic index behave in a hydrologically similar way.

## GLUE METHODOLOGY

The GLUE methodology was developed in response to the recognition that model predictions are necessarily uncertain, due to the problems of model and system identification and data inaccuracy/unavailability (Beven 1989, Beven & Binley, 1992). The GLUE method aims to estimate the uncertainty which exists in model predictions given the available data by the use of Monte Carlo simulation. GLUE requires values of a likelihood measure (as detailed below) for parameter sets within the model, which reflect how well that parameter set fits the available data. Parameter sets are randomly chosen from uniform (or log uniform where appropriate) prior ranges between limits which are chosen on the basis of physical reasoning, prior modelling experience and field data where available. The parameter ranges used in this study are shown in Table 1.

TABLE 1. *Parameter ranges used in Monte-Carlo Analysis*

| Parameter | Units | Minimum | Maximum | Description |
|---|---|---|---|---|
| SZF | $m^{-1}$ | 1 .0 | 25.0 | Groundwater/flow recession parameter |
| AK0 | $m.hr^{-1}$ | 0.01 | 3.0 | Saturated surface conductivity |
| Te | $m^2.hr^{-1}$ | 0.01 | 30.0 | Effective saturated profile transmissivity |
| DTH1 | — | 0.01 | 0.25 | Soil moisture content between field capacity and saturation |
| SRMAX | m | 0.001 | 0.30 | Maximum soil root zone depth |

In the case of no *a priori* knowledge all parameter sets within the specified ranges are given equal likelihood weights. In this study posterior likelihoods are derived from the calibration period performance. These then become prior likelihood distributions for the validation periods. Goodness of fit of each simulation was assessed based on the Nash & Sutcliffe (1970) flow efficiency measure ($R^2$), which is equivalent to a coefficient of determination. From the 10,000 realisations of the model for each period a number of realisations were chosen as "behavioural" (in terms of providing an acceptable simulation) on the basis of a subjectively chosen $R^2$ threshold. A likelihood measure for the $j^{th}$ of n behavioural realisations is then calculated as:

$$L_i = \frac{R_j^2}{\sum_{j=1}^{j=n} R_j^2} \qquad (2)$$

This ensures that the sum of the likelihood weights for the behavioural simulations is equal to one. All non-behavioural or rejected simulations are given a likelihood of zero. Uncertainty bounds were determined as the 5th and 95th percentiles of the likelihood weighted flow distribution over all behavioural simulations. It should be noted that these are calculated independently for each timestep so that the uncertainty bounds may be composites of many model prediction rather than the results of single model runs.

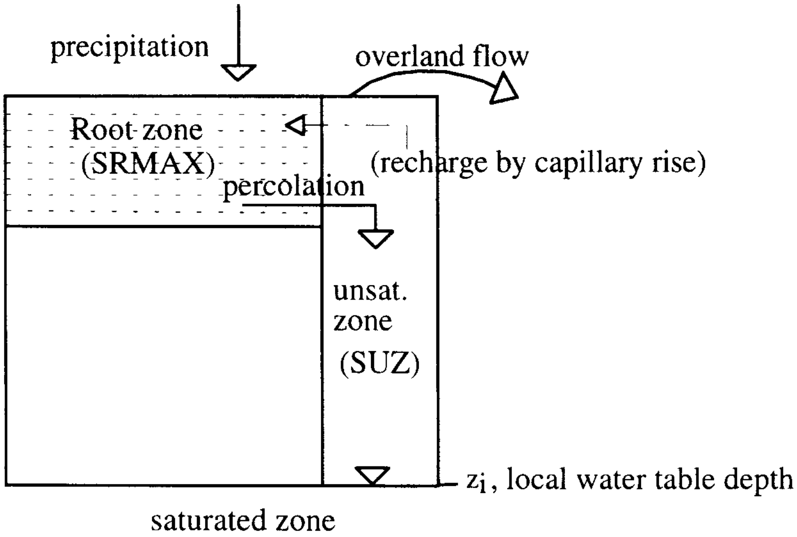

FIG. 3.
Schematic diagram of local soil water stores

## RESULTS

The calibration period ran for 2260 hours from 19th November 1990 to 22nd February 1991 and was chosen to include both low and high flows. Fig. 4 shows the posterior uncertainty bounds for this period conditioned on flow likelihoods from the same period. This likelihood distribution was then used to generate uncertainty bounds for the three validation periods: the preceding winter period of 2600 hours from 16th November 1989 to 4th March 90 (period 1), the summer period of 3700 hours from 17th June 1990 to 18th November 1990 (period 2), and the following winter/spring period of 1432 hours from 22nd Feb. 1991 to 21st April 1991 (period 3). The simulated discharges for these periods are shown in Fig. 5.

It can be seen that observed flows are bracketed by the uncertainty bounds for most of the time. However, it appears that the new root zone algorithm in this version of TOPMODEL has a tendency to produce excessive amounts of overland due to rapid recharging of the near surface storage resulting in overprediction of saturation excess flow at the expense of a slow baseflow response. This may also result from the assumption of a uniform transmissivity function within the catchment, leading to predicted water tables often higher than those observed in boreholes in the catchment, particularly on the divides. However, student exercises with a simpler form of TOPMODEL suggest that improved simulations are possible without the new root zone component, at least once the catchment has wetted at the end of the summer period.

The pronounced seasonality exhibited by the Slapton wood catchment means that it is not as straightforward to achieve a consistent model structure which models separate periods of data as the results shown in Figures 4 and 5 indicate. Table 2 shows that when parameter sets are applied to each period in succession and rejected for not achieving a satisfactory threshold the number of 'behavioural' parameter sets diminishes rapidly, and

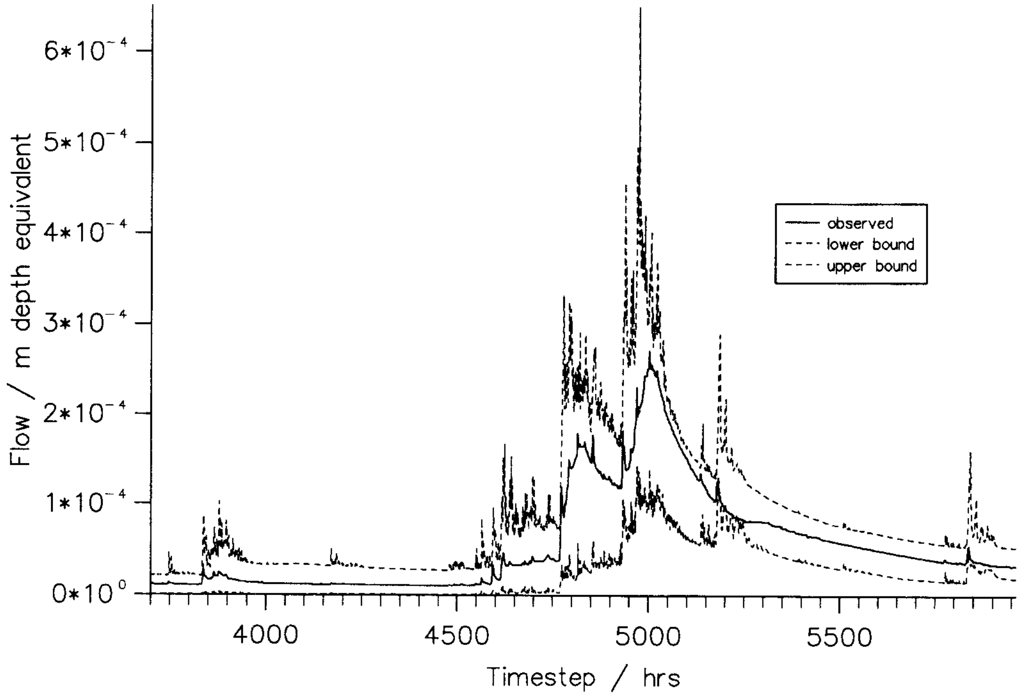

FIG. 4
Uncertainty bounds for calibration period

that no model fits with an R≤ > 0.5 were achieved for the summer validation period (period 3) using this model.

TABLE 2. *Number of model runs over threshold for successive periods*

| Period | Threshold | | | | |
|---|---|---|---|---|---|
| | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| calibration | 237 | 156 | 101 | 43 | 3 |
| validation: period 1 | 109 | 64 | 15 | 0 | 0 |
| period 2 | 102 | 52 | 7 | 0 | 0 |
| period 3 | 0 | 0 | 0 | 0 | 0 |

## CONCLUSIONS

This application of a version of TOPMODEL to the Slapton Wood catchment has shown that after a short calibration period, reasonable simulations of the catchment discharge are obtained for other periods of data. The modified model does, however, overpredict the contributing areas within the catchment. It is suggested that this is due

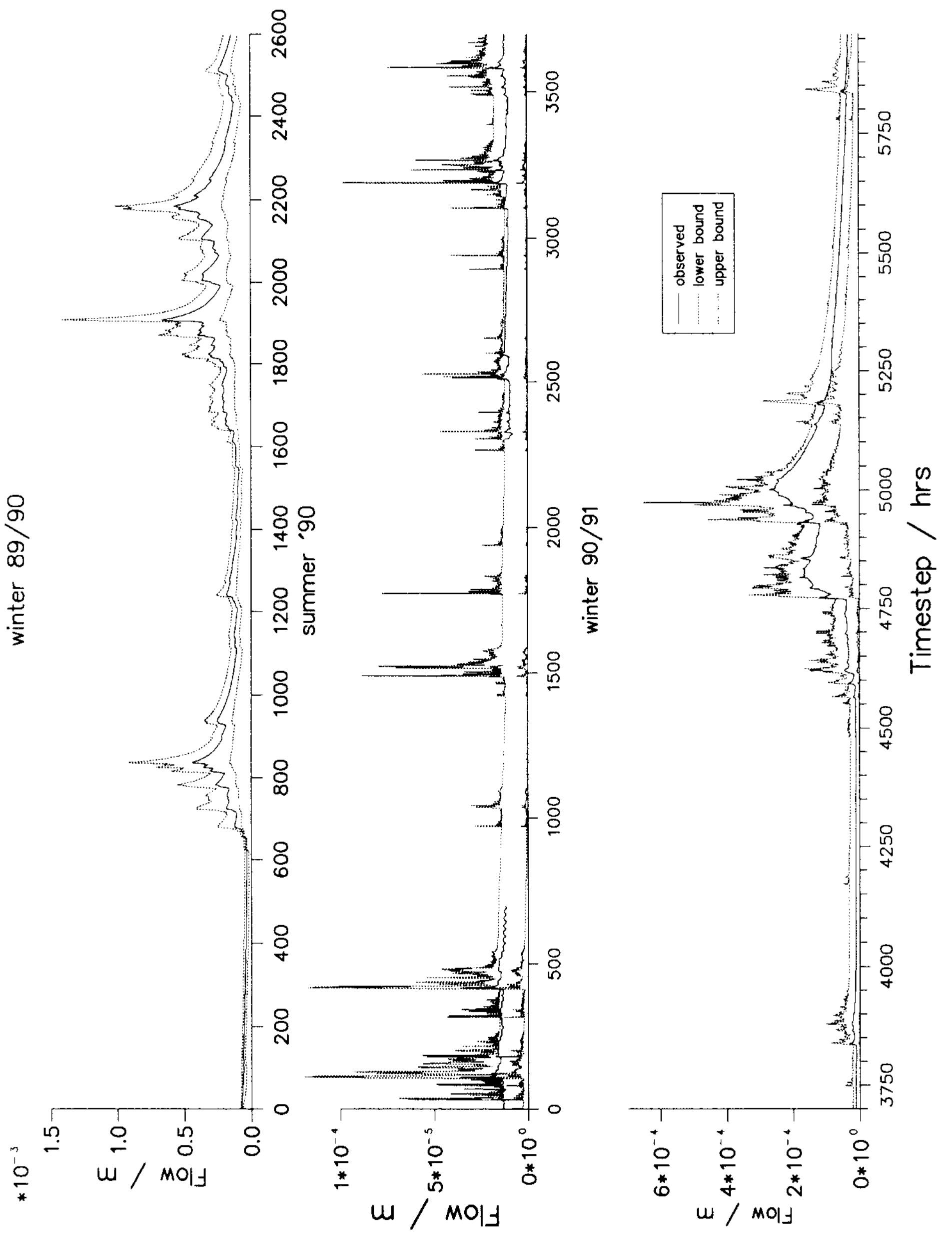

FIG. 5
Prior uncertainty bounds for validation periods

to the modified root zone component retaining too much water in the near surface zone. Further work, reported in Fisher (1995), has attempted to use both remotely sensed and surface soil moisture, and measured water table levels to improve the simulations. The additional data were found to give very little improvement in the flow predictions, because of the complexity of the site and the consequent difficulty of modelling internal states within a relatively simple modelling framework.

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